

DETERMINATION OF FLOW PARAMETERS ON INJECTION OF WATER INTO COMPRESSED AIR

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We examine the state of a medium after injection of water into compressed air and we present a method of evaluating the flow parameters. The results are illustrated with tables and graphs.

In industrial practice we encounter processes in which spray nozzles are used to inject a liquid into compressed air to lower the flow temperature, to raise the moisture content, or to form a two-phase medium. In such an event it is extremely important to determine the mixture parameters at specific points of the duct (the moisture-content ratio, dispersion, concentration, pressure, and the temperature of each of the phases). For example, injection of water may be called for in gas-turbine compressors and similar installations (centrifugal compressors supplying gas for industrial purposes) to achieve compression close to the isothermal, or for purposes of cooling air. We are interested in determining the rates for the cooling of water in air (cooling towers). In studying turbine grids made up of blades designed to function in moist vapor or gas, including a liquid phase, we also frequently employ an air-water mixture containing a specific percentage of moisture and exhibiting the required dispersion.

In this article we will examine the change in the temperature of water and air with the passage of time and we will present a method for an approximate determination of the degree of flow saturation  $\varphi$ , the vapor content  $d$ , and the quantity of evaporated moisture  $G_{ev}$  on the basis of experimental data. We assume that the exchange of heat between the drops and the air is accomplished exclusively by heat conduction and that for the determination of the evaporation rate we can employ the Maxwell equation. This is suitable for a range of temperatures  $t_g \leq 100-150^\circ \text{C}$  and dimensions of

less than 1 mm for the injected drops whose shape is assumed to be spherical.

Let us examine the balance of heat on a drop.

The change in the heat content  $dQ_d$  of the drop results from the expenditure of heat  $dQ_{ev}$  on evaporation and from the change in the heat content  $dQ_g$  of the air:

$$dQ_d = dQ_g + dQ_{ev} \quad (1)$$

We can also write

$$dQ_d = C_w G_d dt_d \quad (2)$$

Under these conditions, as is well known, the heat-transfer coefficient  $\alpha$  is given by

$$\alpha = \frac{\lambda}{r_d}$$

The quantity  $dQ_g$  is then given by

$$dQ_g = \frac{\lambda}{r_d} F_d (t_g - t_d) d\tau \quad (3)$$

According to the Maxwell equation, for a spherical drop [2] we have

$$q = 4\pi r_d \rho (\gamma_{v,d} - \gamma_{v,g})$$

Thus,

$$dQ_{ev} = 4\pi r_d \rho D (\gamma_{v,d} - \gamma_{v,g}) d\tau \quad (4)$$

The function  $\gamma(t)$  can be presented [3], in approximate terms, in the form

$$\gamma = 0.58 \left( \frac{t}{100} \right)^4 (\gamma \text{ in kg/m}^3, t \text{ in } ^\circ\text{C}).$$

Substituting (2), (3), and (4) into Eq. (1), we obtain

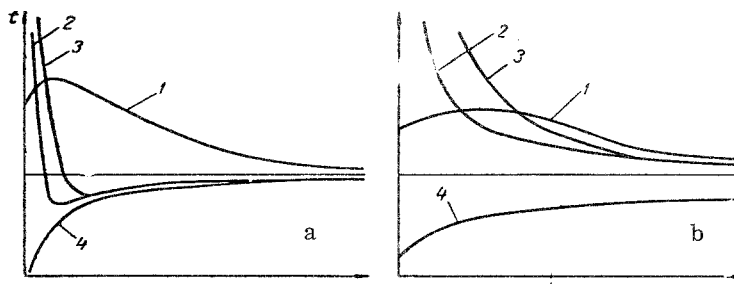


Fig. 1. Change in temperature for air  $t_g$ , drop  $t_d$ , and saturation  $t_{v.g}$  as a function of time during "hot" water injection into flow.  
1)  $t_g$ ; 2)  $t_d(r_{d1})$ ; 3)  $t_d(r_{d2})$ ; 4)  $t_{v.g}$ .

Table 1  
Rates of Cooling for Drops in an Air Flow (time in sec, required to reduce the drop temperature from  $t_{d0} = 90^\circ\text{C}$  to  $t_d$ )

$r_d, \mu$	$t_{v,g} = 10^\circ\text{C}$ for $t_d, ^\circ\text{C}$			$t_{v,g} = 40^\circ\text{C}$ for $t_d, ^\circ\text{C}$		
	20	15	12	50	44	40.4
1	0,00018	0,00043	0,00100	0,000010	0,000022	0,000056
10	0,018	0,043	0,100	0,0010	0,0022	0,0056
100	1,8	4,3	10,0	0,10	0,22	0,56

$$\frac{dt_d}{d\tau} = -\frac{3}{c_{w/d}^2 \gamma_w} \left\{ 0,58 rD \right. \\ \times \left[ \left( \frac{t_d}{100} \right)^4 - \left( \frac{t_{v,g}}{100} \right)^4 \right] - \lambda (t_g - t_d) \left. \right\}. \quad (5)$$

$$t_d - t_{v,g} \approx \frac{\lambda (t_g - t_d)}{4 \cdot 0,58 rD} \left( \frac{100}{t_{v,g}} \right)^4, \quad (7)$$

For an approximate evaluation of the heat-transfer process involving the phases of the flow we can assume that the quantities in formula (5), with the exception of  $t_d$ , are independent of time.

Let us consider the case in which the injected water, in comparison with the compressed air, is of a higher temperature ("hot" water). If the temperature of the injected air is higher than the saturation temperature (Fig. 1), which corresponds to the air pressure, the drop will cool in the air almost instantaneously to the saturation temperature [4]. Any further cooling will proceed in accordance with Eq. (5).

Evaluation of the terms in Eq. (5) shows that the second term in this case is negligibly small. If we neglect that term, when  $\tau_0 = 0$  the solution will have the form

$$\tau = \frac{1}{4bt_{v,g}^3} \left( \ln \frac{t_d + t_{v,g}}{t_d - t_{v,g}} + 2 \arctg \frac{t_d}{t_{v,g}} \right) \Big|_{t_{d0}}^{t_d}, \quad (6)$$

where

$$b = \frac{1,74 rD}{c_{w/d}^2 \gamma_d 100^4}.$$

Table 1 shows the rates for the cooling of drops in a flow of air. The time required to lower the temperature of the drops to values close to the saturation temperatures  $t_{v,g}$  of the vapors in air is presented for the cases in which  $t_{v,g} = 10^\circ\text{C}$  and  $t_{v,g} = 40^\circ\text{C}$ . The drops were assumed to be of various sizes. We see from the table that the temperature  $t_d$  for drops of small dimensions approaches more rapidly the saturation temperature of the vapors. This process is all the more intense, the higher  $t_{v,g}$ . At low saturation temperatures  $t_{v,g}$  a greater amount of time is needed for cooling. The change in drop dimensions due to evaporation to the point at which the drop temperature is close to the saturation temperature did not exceed 2% in this case.

After the drop temperature fell close to the saturation temperature, the change in the heat content  $dQ_d$  of the drop relative to  $dQ_{ev}$  and  $dQ_g$  is small, since the drop temperature is not significantly altered by the quantity of evaporated moisture.

It then follows from Eq. (5) that for  $dt_d/d\tau \approx 0$  the relationship between  $t_d$ ,  $t_g$ , and  $t_{v,g}$  will be of the following form at any given instant of time:

i. e., at higher saturation temperatures the drop temperature is only slightly greater than the saturation temperature of the vapors in air. After the drop temperatures have been leveled off to the temperature determined from Eq. (7), evaporation of the drops continues as a result of the heat from the air.

Let us determine, in approximate terms, the cooling rates for air in a mixture. Within a time  $d\tau$ , let the air alter its enthalpy by

$$dQ_g = -c_p dt_g,$$

If the flow contains  $n(r_d)$  drops with radii  $r_d - r_d + dr_d$ , with consideration of (3) we obtain

$$-\frac{dt_g}{t_g - t_d} = \frac{4\pi\lambda}{c_p} \int_0^\infty n(r_d) r_d dr_d d\tau. \quad (8)$$

Having equated the boundary conditions  $\tau_0 = 0$  and  $(t_g - t_d)\tau_0 = (t_g - t_d)_0$  when  $t_d = \text{const}$  and  $n(r_d) = \text{const}$ , we have

$$\begin{aligned} & t_g - t_d \\ &= (t_g - t_d)_0 \exp \left[ -\frac{4\pi\lambda\tau}{c_p} \int_0^\infty n(r_d) r_d dr_d \right]. \end{aligned} \quad (9)$$

To understand the rate of cooling for air we might note that the time required to reduce the difference  $t_g - t_d$ , for example, by a factor of two with a moisture content of  $y = 1\%$  in the assumption that the flow contains drops of only a single radius, for  $r_d = 10 \mu$  amounts to  $\tau = 0.2$  sec, and for  $r_d = 100 \mu$  it amounts to  $\tau = 20.0$  sec. If the quantity of moisture in the flow is greater, the calculated time will diminish in proportion to  $y$ .

The tentative curves showing the change in the drop temperature  $t_d$ , in the air temperature  $t_g$ , and in the saturated-vapor temperature  $t_{v,g}$  as a function of time in Fig. 1 illustrate Eqs. (6), (7), and (9) for the case of injection of water into air with a temperature of  $t_{v,g\infty} \geq 30^\circ\text{C}$  (Fig. 1a) and  $t_{v,g\infty} \leq 30^\circ\text{C}$  (Fig. 1b). The drop temperature  $t_d$  is given for two drop dimensions ( $r_{d1} < r_{d2}$ ).

Let us also consider the case of the injection of water with a temperature lower than that of the air ("cold" water).

If the temperature of the injected water is lower than the vapor-saturation temperature, in the initial instant after injection the drops are heated as a con-

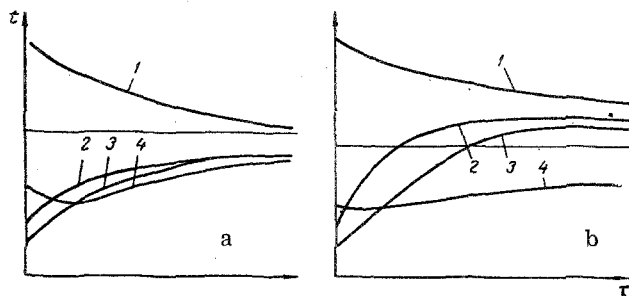


Fig. 2. Change in temperature for air  $t_d$ , drop  $t_g$ , and saturation ( $t_{v.g}$ ) as a function of time during "cold" water injection into flow: 1)  $t_g$ ; 2)  $t_d(r_{d1})$ ; 3)  $t_d(r_{d2})$ ; 4)  $t_{v.g}$ .

sequence both of the condensation of water vapor on the drops and as a result of the heat from the air. A comparison of each of the terms in Eq. (5) for the chosen example with a vapor-saturation temperature of  $t_{v.g\infty} > 30-35^\circ\text{C}$  shows that the rise in the drop temperature results primarily from vapor condensation (Fig. 2a). In this case, the drop temperature is found from Eq. (6). The results obtained in the calculation of the time required to heat various drops from  $t_d = 10^\circ\text{C}$  to the instantaneous values of  $t_d$  when  $t_{v.g} = 40^\circ\text{C}$  are given in Table 2 for this case. We see from the tables that the water injected into the flow is rather quickly raised to the vapor-saturation temperature in the air. The further changes in the water temperature  $t_d$  and in the air temperature  $t_g$  are determined from Eqs. (7) and (9).

When "cold" water (Fig. 2b) is introduced into a flow with  $t_{v.g\infty} < 30-35^\circ\text{C}$ , vapor condensation can be neglected as small in comparison to the exchange of heat with the air and Eq. (5) assumes the form

$$\frac{dt}{d\tau} = \frac{3\lambda}{c_{w,d}^2 \gamma_w} (t_g - t_d). \quad (10)$$

If the quantity of injected water is small, when the drops are heated by the heat from the air, we can assume  $t_g$  to be constant. Thus, having solved Eq. (10) for  $\tau_0 = 0$ , we have

$$t_g - t_d = (t_g - t_d)_0 \exp\left(-\frac{3\lambda\tau}{c_{w,d}^2 \gamma_w}\right). \quad (11)$$

For example, to reduce the temperature difference  $t_d - t_g$  by a factor of two, for drops with dimensions of  $r_d = 10 \mu$  we need a period of time equal to  $\tau = 0.35 \cdot 10^{-2}$  sec, while for dimensions of  $r_d = 100 \mu$  the time

must be  $\tau = 0.35$  sec. In the following instants of time, a relationship is developed between the temperature of the water, air, and saturation, which is determined from formulas (7) and (9).

From the over-all pattern of physical phenomena accompanying this process we can determine the actual degree of saturation  $\varphi$  and the vapor content  $d$  of the mixture at the instant of time  $\tau$  in addition to the quantity of evaporated moisture. For this we will first evaluate the parameters of complete saturation in the compressed air. In the is-diagram for moist air [5], from the known degree of saturation  $\varphi_A$  of the outside air and from the barometric pressure  $P_b$  we find the saturation pressure  $P_A$ . Further, from  $P_A$  and the outside-air temperature  $t_A$  we determine in the is-diagram the point A which corresponds to the parameters of the outside air (Fig. 3). If we assume that the vapor content did not change after compression in the compressor and after cooling of the air in the cooling mechanism and in the tubing, we can determine point H by measuring the dry-air temperature  $t_H$  before mixing. The dry-air saturation ratio  $\varphi_{c.g}$  prior to mixing is given by

$$\varphi_{c.g} = \frac{P_H}{P_{c.g}}.$$

Without considering the heat of the injected water, we determined point C corresponding to the air parameters after mixing from  $i_H$  and from  $P_{c.g}$ , with the assumption that  $\varphi = 1$ .

We will use successive approximations to consider the heat of the injected water.

In first approximation

$$\Delta i_1 = \frac{G_f}{G_g} (i_f - i_H).$$

Table 2  
Time Required to Heat Drops in an Air Flow from  $t_{d0} = 10^\circ\text{C}$  to the Instantaneous Values of  $t_d$  When  $t_{v.g} = 40^\circ\text{C}$  as a Function of the Drop Radius  $r_d$

$r_d, \mu$	$t_d = 20^\circ\text{C}$	$t_d = 30^\circ\text{C}$	$t_d = 35^\circ\text{C}$	$t_d = 38^\circ\text{C}$
1	$0.2 \cdot 10^{-4}$	$0.4 \cdot 10^{-4}$	$0.58 \cdot 10^{-4}$	$0.76 \cdot 10^{-4}$
10	$0.2 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.58 \cdot 10^{-2}$	$0.76 \cdot 10^{-2}$
100	0.20	0.40	0.58	0.76

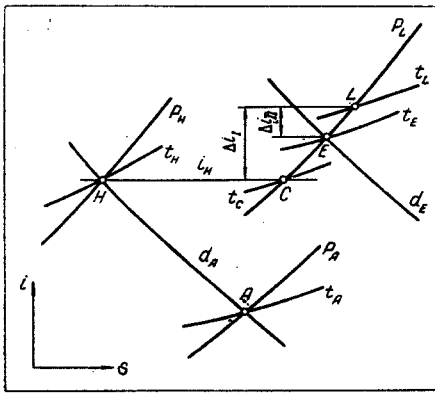


Fig. 3. Determination of air vapor content in is-diagram.

The point L is separated from point C along the isobar  $P_L$  by  $\Delta i_I$ .

In second approximation

$$\Delta i_{II} = \frac{G_f}{G_g} (i_{wc} - i_{wL}).$$

We find point E in similar fashion.

In those cases it is sufficient to have two approximations, and we can take the parameters at point E as those for the complete saturation of the flow. If the water is injected into uncompressed air, point C will coincide with point A.

The actual degree of saturation  $\varphi$  can be found from the temperature  $t_E$  and from the readings of thermocouples in the air-water mixture. The thermocouple in the flow of moist air measures the average temperature  $t_{av}$  of drops exhibiting various radii; when  $t_{v, g\infty} > 30^\circ \text{C}$  this average temperature is close to the saturation temperature  $t_{v, g}$ .

If  $t_{v, g\infty} < 30^\circ \text{C}$ , for "hot" injected water the temperature  $t_{av}$  may be higher than  $t_g$  (Fig. 1b), while for "cold" water it is  $t_{av} < t_g$  (Fig. 2b). With consideration of the function  $\gamma(t)$  and Eq. (7) we can write

$$\varphi = \left( \frac{t_{v, g}}{t_E} \right)^4 \approx \left( \frac{t_d}{t_E} \right)^4 \left[ 1 - \frac{\lambda(t_g - t_d)}{0.58 r D t_E} \left( \frac{100}{t_E} \right)^4 \right]. \quad (12)$$

Correspondingly, the actual vapor content will be

$$d = \varphi d_E \frac{P_L - P_E}{P_L - P_{v, g}}.$$

We find the quantity  $G_{ev}$  from

$$G_{ev} = G_g (d - d_A).$$

If  $t_{v, g\infty} < 30^\circ \text{C}$ , the quantity of water evaporated as a result of the heat from the air can be neglected.

The above-presented analysis of the physical processes in the air-water mixture is confirmed by experimental materials obtained on a test stand.

The working fluid was prepared for the experiments on the test stand by injection of water through centrifugal spray nozzles. The quantity of injected water was altered by the switching on of the required number of spray nozzles. The average time required for the passage of the flow from the instant of injection to the point at which the measurements were to be carried out amounted to 0.1–0.5 sec and the modal radius of the drops was  $30 \mu$ .

Table 3 shows the results obtained in measuring the temperature  $t_H$  of the air ahead of the metering disk, the temperature  $t_{av}$  of the air-water mixture, the total-saturation temperature  $t_{v, g\infty}$ , and the temperature  $t_f$  of the injected water. Comparison of  $t_{av}$  and  $t_{v, g\infty}$  confirms the calculation results. For  $t_{v, g\infty} < 30^\circ \text{C}$  the drops of water cannot cool off and  $t_{av} > t_{v, g\infty}$  for  $t_{v, g\infty} > 30^\circ \text{C}$ , although we have  $t_{av} \approx t_{v, g\infty}$ , but the temperature difference is not greater, since the air-water mixture succeeds in approaching the saturation temperature. Some deviations in the table from the indicated quantitative relationships are explained by the different vapor contents in the air prior to mixing and by the various stay times of the drops in the flow prior to entry into the measuring cross section.

#### NOTATION

$\varphi$  and  $d$  are the degree of saturation and the vapor content at a particular instant of time;  $P_{c, g}$  and  $\varphi_{c, g}$  are the pressure and degree of saturation for the compressed air prior to mixing;  $G_d$ ,  $r_d$ ,  $F_d$ , and  $t_d$  are the mass, the radius, the surface, and the temperature of the drop;  $t_{d0}$  is the temperature of the drop at the initial instant of time;  $G_{ev}$  is the quantity of evaporated moisture;  $q$  is the evaporation rate;  $c_w$  and  $\gamma_w$  are the heat capacity and specific weight of the water;  $\lambda$  is the thermal conductivity of the air;  $\gamma$  is the specific weight of the saturation line;  $\gamma_{v, d}$  is the concentration or specific weight of the vapor in equilibrium with the drop (near the drop), and it corresponds

Table 3

Values (in  $^\circ \text{C}$ ) for the Air Temperature  $t_H$ , the Injected Water  $t_f$ , the Mixture  $t_{av}$ , and the Saturation Temperature  $t_{v, g}$  in the Experiments

$t_H$	26	26	28	23	24	21	31	34	31	29	31	41	43
$t_f$	82.5	81	80	84	91	90	86	87	86	76	83	96	88
$t_{av}$	26	28	27	28	38.5	31	32	31	31	36	36	38	40
$t_{v, g\infty}$	16	23	24	25	27.5	28	31	33	33	34	35	41	43

to the temperature  $t_d$ ;  $\gamma_{v.g}$  is the concentration or specific weight of the vapors in the air;  $t_{v.g}$  is the saturation temperature corresponding to  $\gamma_{v.g}$ ;  $t_{v.g^\infty}$  is the saturation temperature when the temperatures have been completely leveled off;  $\tau$  is the time;  $\tau_0$  is the initial instant of time;  $D$  is the diffusion coefficient;  $r$  is the latent heat of evaporation;  $t_g$ ,  $P$ ,  $c_p$ , and  $G_g$  are, respectively, the temperature, the pressure, the heat capacity, and the air flow rate;  $n(r_d)$  is the distribution function for the number of drops along the radius;  $\Delta i$  is the enthalpy difference;  $i_f$  and  $G_f$  are the heat content and the quantity of injected water;  $i_{wL}$  and  $i_{wC}$  are the heat content of the water with temperature  $t_L$  and  $t_C$ ;  $t_{av}$  is the average temperature for drops of various radii.

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